GEOGEBRA AS AN INSTRUCTIONAL TOOL TO PROMOTE STUDENTS’ OPERATIONAL AND STRUCTURAL CONCEPTION OF FUNCTION

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Abstract: The contemporary literature acknowledges two sorts of conceptions associated with the function concept: operational conception of function and structural conception of function. The former entails interpreting a function as a dynamic process transforming every input to an output. The latter is attained through constant reflection upon a function process which eventually leads to encapsulation or reification of that process into a unified entity. Structural conception enables one to use a function in further processes as if it was a single object, such as using a function in the processes of derivatives and integrals. It is in this respect students need to possess a structural conception of function so that they could succeed in learning advanced calculus topics including limits, derivatives and integrals. In this paper we discuss the opportunities that the GeoGebra offers to promote students’ structural conception of function.

Key words: Function concept, process conception of function, object conception of function, graphical and algebraic representations, GeoGebra software.

INTRODUCTION

The concept of function has often been used as an organising principle in the teaching of mathematics (Yerushalmy & Schwarz, 1993). Simply defining function is a relation (a process) that matches (transforms) inputs to outputs. It has two fundamental properties: univalence and arbitrariness conditions. The univalence aspects suggest that every element in the domain must be assigned to a unique element in the co-domain while the arbitrariness rules out attributing a mechanical rule, algebraic or otherwise, a function. The central importance of the function concept in mathematics prompted interest in studying the cognitive processing of the functions. These studies indicated that most students from elementary level to undergraduate studies have enormous difficulties in understanding the concept of function. Some students wrongly believe that a function is a one-to-one correspondence (Dubinsky & Harel, 1992) while some others indicate strong tendency to think of a function as arithmetic or algebraic formulas (Sfard, 1992). Students’ mental images of function are largely restricted to smooth and continues line or curve. This misconception causes students to reject the graphs of functions in strange shapes but to accept the graphs of equations which are in nice shapes, such as a graph of a circle, as a graph of function. Most students lack an ability to shift between algebraic and graphical representations of the same function. For instance, in Even’s (1998) research only 14% of 152 prospective teachers were able to move from an algebraic to a graphical from of a quadratic function so that they could work out the number of solutions that the corresponding quadratic equation had. Still some others indicate a lack of understanding of the sub-concepts of the functions. Tall & Bakar (1992) reported that half of the university students in their sample rejected the graph of a constant function because these
students possessed a misconnection that a function has variables \( x \) and \( y \); and when \( x \) changes accordingly \( y \) should change.

A review of available literature acknowledges two sorts of conceptions concerning the idea of function: an operational (action-process) conception of function and a structural (object) conception of function (Sfard, 1992; Dubinsky & Harel, 1992; Breidenbach et al, 1992). The former entails interpreting a function as a process transforming every input to an output (ibid). A process is a sequence of actions; therefore it is dynamic in nature. Nevertheless, an operational conception could be considered at two levels in accord with the quality of understanding that the students display when dealing with a function process. Some students may need an explicit algebraic formula so that they insert element(s) into the function and work out its image through step-by-step physical or mental calculations. Dubinsky and his colleagues consider this sort of understanding as an action conception of function (see, for instance, Dubinsky & Harel, 1992; Breidenbach et al, 1992). A process conception, which is considered to be at a higher level of sophistication in the continuum of an operational conception, is attained through interiorising actions associated with the previous step. The possessors of a process cooption are able to think of a function process in terms of inputs ad outputs without necessarily making physical or mental manipulations. Once attained a process can be reversed or combined with other processes (Cottrill et al, 1999); for instance, this quality of understanding enables one to combine two constant functions or to interpret a graph of a function point-by-point. Constant reflections upon a function process would eventually lead to its encapsulation (Dubinsky & Harel, 1992) or reification (Sfard, 1992) as an object. A structural (object) conception of function requires interpreting a function as a unified entity in which the process and the properties of the function have been combined. Those who attained a structural conception of function could use a function in the processes of derivatives and integrals as if it was a single entity. They could deal with a graph of a function in a global way without necessarily employing point wise approaches.

So, what does GeoGebra offer to support students’ development of the function concept? As illustrated before, initial stage of operational conception of function, so called an action conception of function, entails an ability to make manipulations with the algebraic expressions – inserting elements into an expression and calculating their images in step-by-step manner. Students would attain this sort of manipulative skills in the traditional teaching-learning environments. Traditional approaches may also promote, to some extent, the development of a process conception. Yet, GeoGebra has still something to offer to strengthen students’ process cooption of function. It is considered that one crucial aspect of a process conception entails the ability to move freely between algebraic and graphical representations of a function. At the process conception level students could link the representations through point-by-point mappings – they concentrate upon the critical features of a graph (e.g., maxima, minima, intersection points) and, then, try to relate them to the corresponding elements in the algebraic expression, or they do the reverse. Using the GeoGebra the teaching-learning community could conduct several activities in this kind. For instance, the graph of function \( f(x)=3x-6 \) can easily be generated using GeoGebra (see Figure 1). Then, the students can be asked to find out what sort of changes they observe on the graphs of function as they replace coefficient and constant terms with different numbers in the algebraic
expression. More specifically they could be asked to investigate the relations between the coefficient and constant terms in the algebraic expression and the points where the graph intersects $x$ and $y$-axis. Inversely, using the slider students can drag up and down the graph and, then, look for the corresponding changes in the coefficient and constant terms in algebraic form.

![Figure 1: Connecting algebraic and graphical representations to promote students’ process conception of function](image1)

The actual benefits of GeoGebra can be seen in promoting students’ structural conception of the functions. As illustrated before a structural conception entails the ability to manipulate a function as if it was a single entity. A graphical depiction of a function incites very much a structural interpretation, because in such a figure the process and the properties of the function concept are unified (Sfard, 1992). In this respect, a meaningful understanding of the translation of functions along the coordinate axis requires a structural conception. GeoGebra allows manipulating this sort of activities and investigating the relations between the changes on the graphical and algebraic depictions. For instance, the teaching-learning community would sketch the graph of $f(x) = \cos x$ (see Figure 2). Then, they would draw the graphs of $g(x) = (\cos x) + 3$ and $h(x) = (\cos x) - 3$ on the same coordinate system.

![Figure 2: Translation of the graph of $f(x) = \sin x$ parallel to the $y$-axis](image2)

The teaching-learning community would find out through collective reflection that the graphs of $g(x) = (\cos x) + 3$ and $h(x) = (\cos x) - 3$ are obtained by translating the graph of $f(x)$ 3 unit along the $y$-axis respectively in the positive and negative directions. The teachers would continue to probe their students to discover the underlying meaning of why the graph of $f(x)$ is translated along the $y$-axes, but not parallel to the $x$-axis.
Another activity to enhance students’ structural conception of function might involve exploring the changes in the general behaviour of a graph of a function in accord with the changes in its algebraic form. The teaching-learning community could sketch the graph of $f(x)=x^2$ and, then, they could change the coefficient of $x^2$ (see Figure 3).

![Figure 3: GeoGebra screen showing an investigation of the changes in the general behaviour of a parabola in accord with the changes in its algebraic form.](image)

It is seen in the above figure that actions taken upon the coefficient of $x^2$ cause changes on the graph of function. GeoGebra allows students to observe this change in a more global way and find out the idea that ‘as the coefficient of $x^2$ increases, the graph of $f(x)=x^2$ gets closer to the $y$-axis; and as the coefficient of $x^2$ decreases, the graph gets flattened’.

Students manipulate a function as if it was a single object when they take its derivative and integral. However, most students carry out such manipulations in a procedural way and very few of them can imagine the impacts of these operations on a function. The dynamic GeoGebra software allows students to observe the changes that a function goes through when its derivative and integral is taken. It is in this respect GeoGebra promotes students’ structural conception of function. For instance, the teaching-learning community could sketch the graph of a third power function, for instance $f(x)=x^3$; and then they could obtain a new function by taking its derivative, $f'(x)=3x^2$, and sketch the graph of it on the same screen. They would find out the impacts of derivative on the function by comparing the general behaviours of the graphs of $f(x)=x^3$ and $f'(x)=3x^2$.

![Figure 4: Graphical connection between a function and its derivative.](image)
CONCLUDING REMARKS

The concept of function is at the heart of mathematics curriculum, and it is often used as an organising principle in the teaching of mathematics (Yerushalmy & Schwarz, 1993). Nevertheless, most students have great difficulties in developing a proper understanding of this notion. The epistemological complexity of the function concept (profusion of properties and the sub-notions of the function) and the diversity of the representations used (e.g., graphs, ordered pairs, and algebraic expressions) are the major factors that would make the subject difficult to understand (Eisenberg, 1991).

If it is to go beyond mere manipulation with the algebraic expressions an understanding of the function concept requires possessing operational (process) and structural conceptions of the function. In this paper we tried to illustrate how dynamic GeoGebra software could be used to promote students’ operational (process) and structural (object) conceptions of the function. It is indicated that GeoGebra allows an integrated use of expressions and graphs and, thus, gives students an opportunity to understand the underlying meaning of rules, procedures and the factual knowledge associated with the notions of functions. GeoGebra promotes students structural conception of function in that it gives them an opportunity to observe the global behaviour of the graph of a function in accord with the changes in its algebraic form. It is illustrated also that a proper use of the GeoGebra allows students to monitor the impacts of derivative and integral operations on a function in the graphical context. Finally, it is indicated throughout the paper that GeoGebra enhances students’ visual ability and enable them to conduct visual strategies to resolve problems related to the concept of function.

In closing, it is worth noting that most of the ideas presented in this paper are theoretically based. These ideas can be validated through an experimental study; and this is the issue for further research.

REFERENCES


